



# Measurement of angular velocity in the perception of rotation

José F. Barraza, Norberto M. Grzywacz \*

*Department of Biomedical Engineering, University of Southern California, University Park, OHE 500, Los Angeles, CA 90089-1451, USA*

Received 30 August 2001; received in revised form 8 July 2002

---

## Abstract

Humans are sensitive to the parameters of translational motion, namely, direction and speed. At the same time, people have special mechanisms to deal with more complex motions, such as rotations and expansions. One wonders whether people may also be sensitive to the parameters of these complex motions. Here, we report on a series of experiments that explore whether human subjects can use angular velocity to evaluate how fast a rotational motion is. In four experiments, subjects were required to perform a task of speed-of-rotation discrimination by comparing two annuli of different radii in a temporal 2AFC paradigm. Results showed that humans could rely on a sensitive measurement of angular velocity to perform this discrimination task. This was especially true when the quality of the rotational signal was high (given by the number of dots composing the annulus). When the signal quality decreased, a bias towards linear velocity of 5–80% appeared, suggesting the existence of separate mechanisms for angular and linear velocity. This bias was independent from the reference radius. Finally, we asked whether the measurement of angular velocity required a rigid rotation, that is, whether the visual system makes only one global estimate of angular velocity. For this purpose, a random-dot disk was built such that all the dots were rotating with the same tangential speed, irrespectively of radius. Results showed that subjects do not estimate a unique global angular velocity, but that they perceive a non-rigid disk, with angular velocity falling inversely proportionally with radius.

© 2002 Elsevier Science Ltd. All rights reserved.

*Keywords:* Optic flow; Motion; Rotation; Speed; Angular velocity

---

## 1. Introduction

Natural motions in the world and ego motion generate complex optical flow fields in the retina. These fields can be a rich source of information for orientation, navigation, and the perception of the three-dimensional world (Gibson, 1950; Koenderink & van Doorn, 1976). Koenderink and van Doorn (1976) have shown that small rigid patches of moving objects produce motion patterns in the image that can be decomposed into five types of motion, including translation, expansion, and rotation. Therefore, one wonders whether these complex motions are processed metrically by the brain, that is, whether it estimates their parameters. This seems to be the case for translation, since humans can discriminate the direction (De Bruyn & Orban, 1988) and the speed (Johnston, Benton, & Morgan, 1999; McKee, 1981) of motion well. However little is

known about whether the other types of motion are estimated metrically by the brain.

For example, what information does the visual system use to evaluate how fast a rotational motion is? One possibility is that the brain computes the speed of rotation by the local estimation of linear velocity, making this computation dependent on the radius of the circular trajectory. Another possibility is that the brain computes angular velocity, a parameter that is harder to obtain, but which would be more useful to estimate the speed of rotation. Unfortunately, there is no agreement on this point yet. Some psychophysical studies suggest that the perceived speed depends on the global pattern of the motion. For instance, the speed of different kinds of motion, such as translation, rotation and expansion, can be perceived as different even if the distributions of their linear components of speed are the same (Bex & Makous, 1997; Geesaman & Qian, 1996, 1998). These results are consistent with the idea that there are specific mechanisms that analyze complex motions metrically (Freeman & Harris, 1992; Morrone, Burr, & Vaina, 1995; Regan & Beverley, 1985). However, Werkhoven

---

\* Corresponding author.

E-mail address: [nmg@bmsr.usc.edu](mailto:nmg@bmsr.usc.edu) (N.M. Grzywacz).

and Koenderink (1993) suggested that the visual system could not use angular velocity, since they found a bias towards linear velocity in the matching of two rotating annuli of different radii. Unfortunately, Werkhoven and Koenderink's results could not be explained by tangential linear-speed discrimination either. Perhaps, angular velocity was not well discriminated in their experiment because their stimuli had few dots and thus provided poor rotational information.

Here, we retook this issue to explore whether there are conditions in which the visual system uses a metric of angular velocity to evaluate rotational motion. Part of our results appeared previously in abstract form (Barraza & Grzywacz, 2001).

## 2. Methods

### 2.1. Stimuli

We used two different kinds of stimuli in this study. The first consisted of two sequentially displayed quasi-random-dot annuli revolving around a fixation mark (Fig. 1a), the reference and the test stimuli. Each dot in the stimulus was randomly located into one of  $N_d$  (number of dots) portions of the annulus, to balance the spatial distribution of the dots. (The variance of the locations of the dots could be zero, yielding a regular annulus such as in Fig. 1a.) The radius of the reference stimulus was  $0.9^\circ$  (except for the experiment where the influence of radius was studied) and its dots revolved with an angular velocity of 1.25 rad/s. To determine the number of dots, the computer used the radius and the experimental variable  $d_{nd}$ , which expressed the mean distance between neighbor dots. The size of the dots was  $5.5'$  and they were displayed with a luminance of  $19.5 \text{ cd m}^{-2}$  on a background whose luminance was  $39 \text{ cd m}^{-2}$ . To avoid transient effects at the time of stimulus

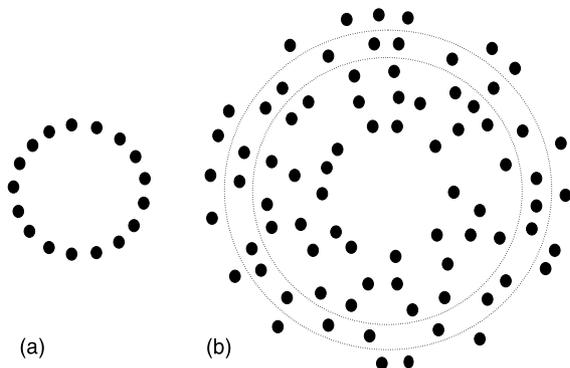


Fig. 1. Examples of the two reference stimuli used in this study. Panel (a) shows the first example, which is an annulus of regularly spaced dots that rotate around a center. Panel (b) shows the non-rigid stimulus in which all the dots rotate with the same tangential speed. In this example, the locations of the dots along the annuli were randomized.

onset as well as motion after-effects, the contrast of the dots was temporally modulated with a Gaussian function with a standard deviation of six frames. The interval between the reference and the test was 16 frames.

In the second stimulus, the reference was a quasi-random-dot disk in which the dots were located in seven concentric annuli. We built each annulus using the same method as that used for the single-annulus stimuli (Fig. 1b). The direction of motion of the dots was consistent with rotation, but their speeds were the same regardless of radius. The percept was of a non-rigid rotation, with the apparent rate of rotation falling with radius. To compare the perceived angular velocity at a given radius with that of the test, two circles were drawn to demarcate the desired annulus. In turn, the test stimulus was the same as in the first experiment.

Stimuli were displayed on a high-resolution CRT monitor at a frame rate of 62 Hz.

### 2.2. Procedures

We performed four experiments to explore whether the human visual system uses angular velocity to evaluate the speed of rotation and to explore what are the parameters that influence its estimation. In all experiments, subjects indicated by pressing a button of the mouse which of the stimuli, first or second, rotated faster. The order of presentation of the reference and test stimuli was random. We used a 2AFC paradigm with the method of constant stimuli to obtain the subjects' psychometric functions. The matching velocity was calculated by fitting cumulative Gaussian curves to these functions. To obtain these functions, a set of six stimuli was used in each of two blocks of trials. Each stimulus appeared a total of 20 times per block.

We defined a bias measure to quantify by how much the matching angular velocity deviated systematically from the reference angular velocity. This bias was expressed as a percentage of the reference angular velocity.

### 2.3. Subjects

Four subjects participated in these experiments, one of the authors and three others naïve as to the purpose of the study. All of them were trained before beginning the experiments. Viewing was binocular, with natural pupils.

## 3. Results

### 3.1. Perceived angular velocity as a function of the test radius

We measured the perceived angular velocity of the test annulus as a function of the test radius ( $R_t$ ) for a fixed value of the reference radius ( $R_r$ ). The test radius

could take values of  $1/2R_r$ ,  $3/4R_r$ ,  $R_r$ ,  $3/2R_r$ , and  $2R_r$ . Both the reference and test annuli had a mean distance between neighbor dots ( $d_{nd}$ ) of  $0.19^\circ$  and thus, the number of dots in the test depended on its radius. In this experiment, the degree of randomization for both the test and the reference was zero, which means that the dots were regularly spaced along the annulus. Fig. 2 shows the perceived angular velocity of the test as a function of the ratio  $R_t/R_r$ . Symbols represent the experimental data, while the dotted line represents the reference angular velocity. The solid line shows the predicted angular velocity for a matching performed by using tangential speed.

The subjects performed the task by using angular velocity. The systematic bias towards tangential speed found by Werkhoven and Koenderink (1993) did not appear in our data. Perhaps, this was because we used a richer rotational stimulus (more dots); our  $d_{nd}$  was  $0.19^\circ$ , whereas theirs was  $1.35^\circ$ . To explore whether the  $d_{nd}$  can account for the differences between the Werkhoven and Koenderink's results and ours, we measured the bias in the perceived angular velocity for a wide range of  $d_{nd}$ .

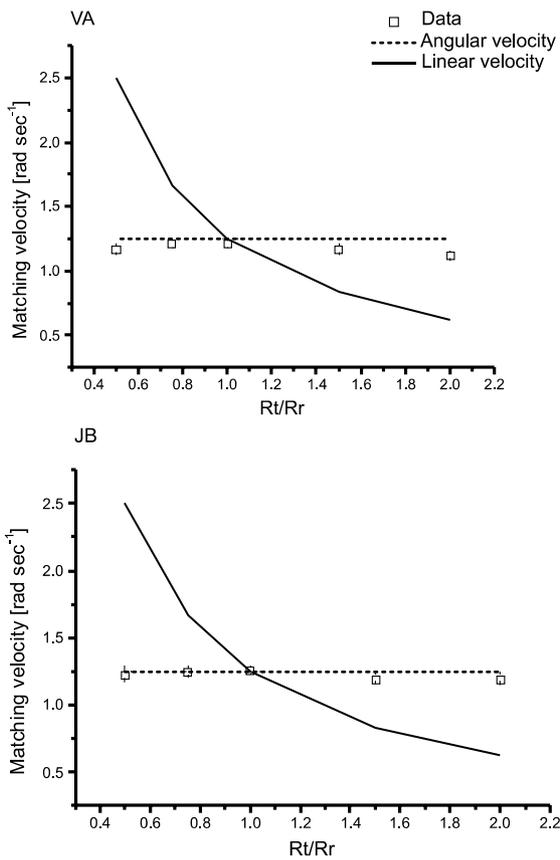


Fig. 2. Perceived angular velocity as a function of  $R_t/R_r$ . Different panels show different subjects. The symbols represent the experimental data, the dotted line represents the angular velocity of the reference, and the solid line represents the angular velocity expected if the task were performed through tangential-speed matching. The results show that the subjects can use angular velocity to match speed of rotation.

### 3.2. The influence of $d_{nd}$ on the bias towards tangential speed

In this experiment, the test radius was half the reference radius and the  $d_{nd}$  varied from  $0.12^\circ$  to  $1.51^\circ$ . Fig. 3

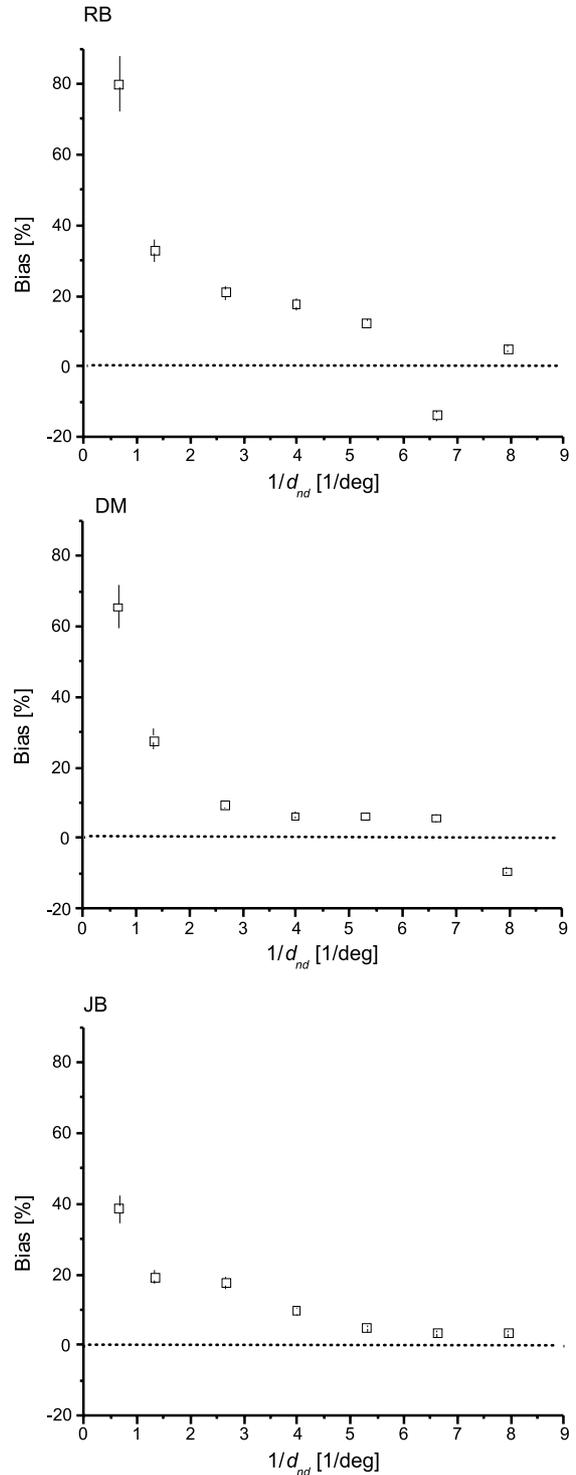


Fig. 3. Bias in the perceived angular velocity towards tangential speed as a function of the inverse of the distance between neighbor dots. The results show that when this distance is large, the bias is high, but that it falls rapidly as distance decreases.

shows the bias as a function of the inverse of  $d_{nd}$ . Results show that, consistently with Werkhoven and Koenderink (1993), there is a bias towards tangential speed for large values of  $d_{nd}$ . However, when the  $d_{nd}$  decreases, which means that the number of dots increases, the bias falls rapidly from approximately 80% to near zero. In other words, when the  $d_{nd}$  is sufficiently small, subjects use angular velocity to perform the task. In contrast, for large values of  $d_{nd}$ , which means poorer rotational signals, subjects also use the tangential speed.

The results of this experiment suggest that the brain can switch between tangential and rotational motion mechanisms according to the incoming retinal data. This switching is not abrupt, but there are conditions in which the perceived angular velocity depends on tangential speed, true angular velocity, or both.

### 3.3. Bias as a function of radius

Next, we measured matching velocity for five values of reference radius to test whether the bias towards tangential speed depends on stimulus size. In this experiment, the test had a radius that could be either half or double that of the reference, that is, for each reference radius, a pair of points were obtained. The  $d_{nd}$  was  $0.38^\circ$  for which a bias of approximately 20% was expected according to the data shown in Fig. 3. The perceived angular velocity is plotted in Fig. 4 as a function of the test radius, along with the angular velocity of the reference (solid line). This figure shows that the magnitude of the bias does not depend systematically on the size of the stimulus (five contiguous points for  $R_t < R_r$  or for  $R_t > R_r$ ).

This result could have some implications on what cells are involved in the process. That the bias does not change for sizes between  $1^\circ$  and  $8.5^\circ$  suggests that angular velocity is processed in receptive fields of at least  $8.5^\circ$ . This means that the motion mechanism underlying this process is not local. Cells in the middle superior temporal cortex may underlie this mechanism, as they can be sensitive to rotation and have large receptive fields (Duffy & Wurtz, 1991; Graziano, Andersen, & Snowden, 1994; Tanaka & Saito, 1989; Tanaka, Fukuda, & Saito, 1989).

### 3.4. Non-rigid stimulus

Finally, we asked whether the measurement of angular velocity required a rigid rotation. Alternatively, we wondered whether the visual system could make independent estimates of angular velocity for different parts of the image. In this experiment, the reference stimulus was a random-dot disk in which the direction of motion of the dots were consistent with rotation, but the speed of all dots was the same regardless the radius. The percept was non-rigid, such that the inner annuli seemed to rotate faster than the outer annuli. This percept

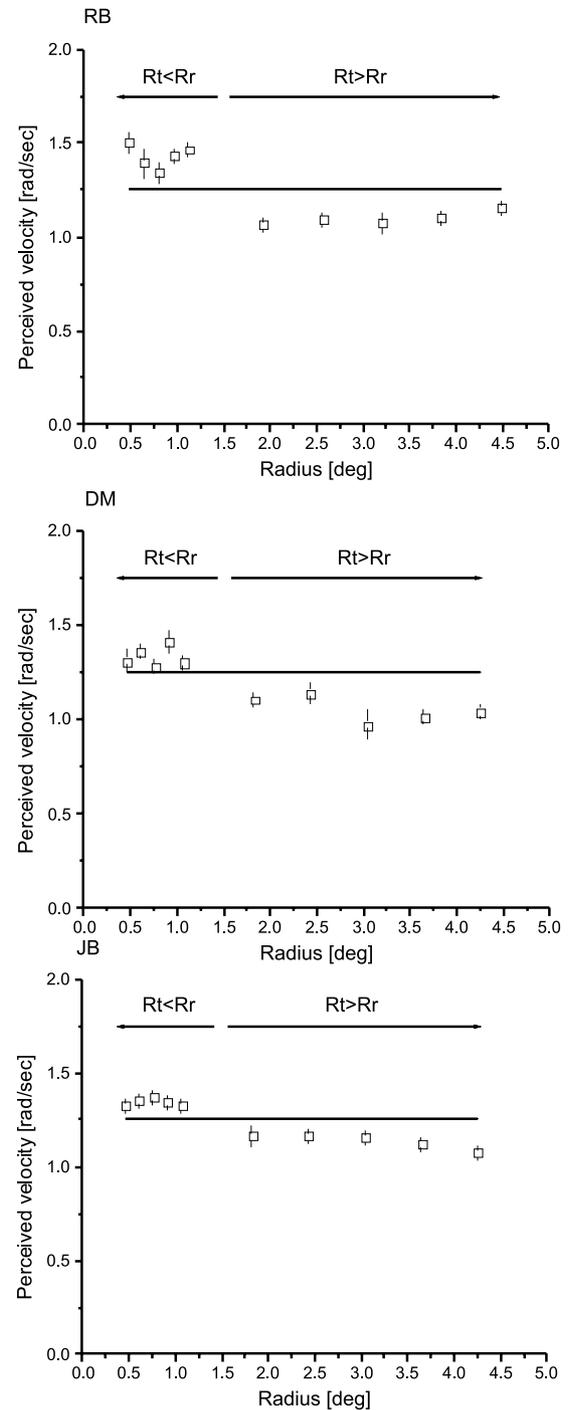


Fig. 4. Perceived angular velocity as a function of the test radius for five values of reference radius. Symbols represent the experimental data and the solid line represents the reference angular velocity. The difference between the data and the angular velocity of the reference is the magnitude of the bias. The results do not show a systematic dependence of the bias on the radius.

supports the finding in Figs. 2 and 3. If the rotational signal is sufficiently rich, then subjects do not use tangential speed directly to perceive how fast dots are rotating. If subjects use tangential speed, then all dots would appear to move equally fast. Another implication

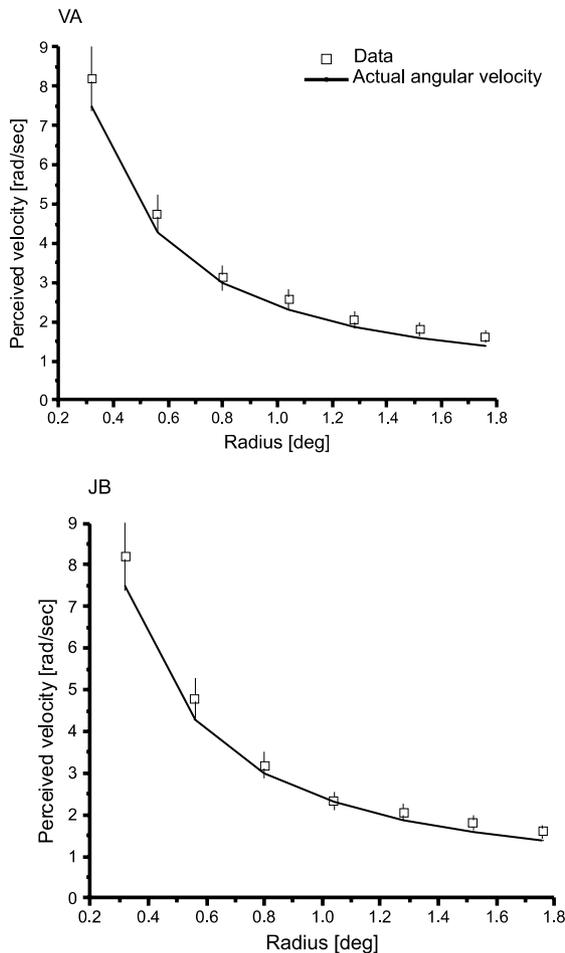


Fig. 5. Perceived angular velocity as a function of radius in the non-rigid disk. Symbols represent the experimental data and the solid line represents the actual angular velocity along the disk. The data match well the actual angular velocities.

of the non-rigid percept was that if subjects measure angular velocity in this display, they must measure several values of it instead of a single global estimate. To explore what information the visual system uses to produce this percept, we measured the perceived angular velocity at each of the concentric annuli. Fig. 5 shows the perceived angular velocity as a function of the radius of the reference disk. Symbols represent the experimental data and the solid lines represent the actual angular velocities in the annuli.

Results show that the perceived angular velocity falls hyperbolically with radius and match well the actual angular-velocity prediction. Therefore, the percept of non-rigidity in this stimulus is due to the brain computing angular velocities independently at different radii of rotation.<sup>1</sup>

<sup>1</sup> However, this explanation does not mean that the stimulus was interpreted as many independent annuli. On the contrary, the circles demarcating the desired annulus did not appear to allow subjects to segment it from the disk.

#### 4. Discussion

We investigated whether the visual system uses angular velocity in the computation of rotational motion. In the first part of the work, we showed that human subjects could match the speed of rotation of two annuli of different radii. This suggests that there is an angular-velocity metric in the computation of speed of rotation. This metric mechanism exists in addition to the translational mechanism shown in other studies (McKee, 1981). Yuille and Grzywacz (1998) have proposed a theory for how the visual system computes simultaneously the parameters of translation, rotation, and other kinds of motion. This motion-coherence theory suggests that the brain fits different models of motion to the incoming retinal data, selecting the best models and their parameters. These fits would not necessarily be global, but rather would be applicable to small local surface patches (Yuille & Grzywacz, 1998). Motions from small, rigid surface patches in the real world can be decomposed into five components (Koenderink & van Doorn, 1976).

Consequently, it is reasonable to believe that the brain includes these motion components as models to analyze complex motions. That the brain can compute angular velocity when faced with a rotation is evidence that there is a specialized metric mechanism for rotational motion, one of these putative components. Further evidence for such mechanisms come from some recent apparent-velocity experiments (Bex & Makous, 1997; Geesaman & Qian, 1996, 1998), which could only be explained by using some form of global-velocity information (Clifford, Beardsley, & Vaina, 1999). In addition, the proposal of such a specialized mechanisms is consistent with psychophysical studies that suggested the existence of looming and rotation detectors (Freeman & Harris, 1992; Morrone et al., 1995; Regan & Beverley, 1985). And several physiological studies have shown that there are cortical cells sensitive to rotation, expansion, and spiral motion (Duffy & Wurtz, 1991; Graziano et al., 1994; Tanaka & Saito, 1989; Tanaka et al., 1989).

However, we also show in this study that subjects do not only use angular velocity to estimate the rate of rotation. They may use tangential speed, too. This is consistent with the results of Werkhoven and Koenderink (1993), who showed that there is a bias in the perceived rate of rotation towards tangential speed. The conditions for which this bias appears are achieved by reducing the number of dots in the annulus.

Why does the number of dots have an effect on the perceived velocity of rotational motion? If one considers, for example, a single dot moving in a circular trajectory, there is no way to know how fast this dot is rotating, unless one observes its motion for a long period. For short periods, the arc that this dot describes

could be part of many trajectories. There are no instantaneous clues in this single-dot stimulus for the brain to assume a rotation. On the other hand, when the number of dots increases, the spatial coincidence of many short motions provides a signal consistent with rotation. Therefore, the probability that the motion is considered a rotation by the brain increases. In other words, as the number of dots increases, the quality of the rotational signal improves, and the brain relies on the rotational model increasingly. Another possible explanation for the effect of the number of dots is that the larger is the number of dots the smaller is the distance between neighbor dots. Hence, the probability that these dots are a part of the same object is higher. This could be used as a cue by the brain to apply global models to retinal data and assume rotation in particular. Whatever prompts the brain to assume rotation more often as the density of the dots increases, this process is likely to be probabilistic. Internal noise would make the signal sometimes cross the threshold for the perception of rotation and sometimes fail to do so. This would explain why the judgment of the rate of rotation appears like using a mixture of angular velocity and tangential speed (Figs. 3 and 4).

The use of angular velocity in a rotational percept is also observed in the non-rigid disk (Fig. 5). Although the tangential speeds are the same over the entire stimulus, subjects perceive the inner annuli rotating much faster than the outer annuli. Moreover, the perceived rate of rotation fits well the actual angular-velocity distribution along the disk. How can one explain this result in terms of the motion-coherence theory? This theory postulates that the brain fits an internal model to the retinal data. Therefore, at a first glance, one would think that a single global angular velocity should be extracted from the non-rigid stimulus. However, this is not the case, as subjects can calculate angular velocities for each annulus independently. Further thought reveals that the motion-coherence theory could explain this result if the brain would trust the retinal data more than the rotational model. This is possible, because this theory has a parameter expressing how much trust there is in the data versus the model. High trust in the data allows the brain to build several estimates of angular velocity instead of a single global one.

### Acknowledgements

We thank the Smith–Kettlewell Eye Research Institute for hosting us during a part of this project. This work was supported by a FOMEC fellowship from the Departamento de Luminotecnia, Luz y Visión, UNT,

Argentina and by the Rachel C. Atkinson fellowship to JFB, and by National Eye Institute Grants EY08921 and EY11170 to NMG.

### References

- Barraza, J. F., & Grzywacz, N. M. (2001). Discrimination of angular velocity in humans. *Investigative Ophthalmology and Visual Science Supplement*, 42, 870.
- Bex, P. J., & Makous, W. (1997). Radial motion looks faster. *Vision Research*, 37, 3399–3405.
- Clifford, C. W. G., Beardsley, S. A., & Vaina, L. M. (1999). The perception and discrimination of speed in complex motion. *Vision Research*, 39, 2213–2227.
- De Bruyn, B., & Orban, G. A. (1988). Human velocity and direction discrimination measured with random dot patterns. *Vision Research*, 28, 1323–1335.
- Duffy, C. J., & Wurtz, R. H. (1991). Sensitivity of MST neurons to optic flow stimuli. II. Mechanisms of response selectivity revealed by small-field stimuli. *Journal of Neurophysiology*, 65, 1346–1359.
- Freeman, T. A. C., & Harris, M. G. (1992). Human sensitivity to expanding and rotating motion: effects of complementary masking and directional structure. *Vision Research*, 32, 81–87.
- Geesaman, B. J., & Qian, N. (1996). A novel speed illusion involving expansion and rotation patterns. *Vision Research*, 36, 3281–3292.
- Geesaman, B. J., & Qian, N. (1998). The effect of complex motion pattern on speed perception. *Vision Research*, 38, 1223–1231.
- Gibson, J. J. (1950). *The perception of the visual world*. Boston, MA: Houghton Mifflin.
- Graziano, M. S. A., Andersen, R. A., & Snowden, R. J. (1994). Tuning of MST neurons to spiral motions. *Journal of Neuroscience*, 14, 54–67.
- Johnston, A., Benton, C. P., & Morgan, N. J. (1999). Concurrent measurement of perceived speed and speed discrimination using the method of single stimuli. *Vision Research*, 39, 3849–3854.
- Koenderink, J. J., & van Doorn, A. J. (1976). Local structure of movement parallax of the plane. *Journal of the Optical Society of America*, 66, 717–723.
- McKee, S. P. (1981). A local mechanism for differential velocity detection. *Vision Research*, 21, 491–500.
- Morrone, M. C., Burr, D. C., & Vaina, L. M. (1995). Two stages of visual processing for radial and circular motion. *Nature*, 376, 507–509.
- Regan, D., & Beverley, K. J. (1985). Visual responses to vorticity and the neural analysis of optic flow. *Journal of the Optical Society of America*, A12, 280–283.
- Tanaka, A., & Saito, H. (1989). Analysis of motion of the visual field by direction, expansion/contraction, and rotation cells clustered in the dorsal part of the medial superior temporal area of the macaque monkey. *Journal of Neurophysiology*, 62, 626–641.
- Tanaka, A., Fukuda, Y., & Saito, H. (1989). Underlying mechanisms of the response specificity of expansion/contraction, and rotation cells, in the dorsal part of the medial superior temporal area of the macaque monkey. *Journal of Neurophysiology*, 62, 642–656.
- Werkhoven, P., & Koenderink, J. J. (1993). Visual size invariance does not apply to geometric angle and speed of rotation. *Perception*, 22, 177–184.
- Yuille, A. L., & Grzywacz, N. M. (1998). A theoretical framework for visual motion. In T. Watanabe (Ed.), *High-Level Motion Processing—Computational, Neurobiological, and Psychophysical Perspectives* (pp. 187–211). Cambridge, MA, USA: MIT Press.